

# Convergence Analysis of Systems with Fixed Point Structure

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Let  $X$  and  $Y$  be two arbitrary sets,  $P \subset X \times Y$  and  $Q \subset X \times Y$ . We consider the problem of finding a couple  $(u, \eta) \in X \times Y$  such that  $(u, \eta) \in P \times Q$ . We prove that, under appropriate assumptions, the existence of the solution of this problem is equivalent to the existence of fixed points of some associated operators  $\Lambda$  and  $\Theta$ . Moreover, the solution is unique if and only if the corresponding fixed points are unique. Then, we move to the framework of metric spaces  $X$  and  $Y$  and, under additional assumptions, we provide necessary and sufficient conditions which guarantee the convergence of an arbitrary sequence  $\{(u_n, \eta_m)\} \subset X \times Y$  to the solution  $(u, \eta)$ . Our abstract results have many applications in the analysis of various nonlinear systems. We illustrate their use in some examples, proving existence and uniqueness results, together with convergence criteria.

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